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Optimal controller design for structural damage detection

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Abstract

The virtual passive control technique has recently been applied to structural damage detection, where the virtual passive controller only uses the existing control devices, and no additional physical elements are attached to the tested structure. One important task is to design passive controllers that can enhance the sensitivity of the identified parameters, such as natural frequencies, to structural damage. This paper presents a novel study of an optimal controller design for structural damage detection. We apply not only passive controllers but also low-order and fixed-structure controllers, such as PID controllers. In the optimal control design, the performance of structural damage detection is based on the application of a neural network technique, which uses the pattern of the correlation between the natural frequency changes of the tested system and the damaged system.

1. Introduction

Vibration-based structural damage detection has received considerable attention in recent years [1–4]. One important and challenging issue is how to use a limited experimental setup, such as a small number of sensors, to detect effectively possible damage. Recently, a transfer function correlation approach [3] was developed for damage detection, based on a comparison of the identified transfer function parameter change and the change of the analytical model due to damage. Rather than a large number of sensors, only a few sensors are required for this approach

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[3,5]. This correlation approach has been applied to identify experimentally the damage location and intensity of a flexible beam system [6].

The vibration-based damage detection method is based on the analysis of the identified parameters, which have uncertainty due to noise and environmental change. In general, the identified mode shapes are more sensitive to noise and environmental uncertainty than the identified natural frequencies. On the other hand, the natural frequencies are sensitive to structural damage, such as stiffness loss and cracking. Thus, the identified natural frequencies are more reliable for damage detection than the identified mode shapes. In real applications the identified natural frequencies of an open-loop system may not provide enough information for damage detection. To generate more information, some researchers have proposed the use of "Twin" structures, where a structure is attached to the tested structure, for damage detection [7]. The concept of physical attachment of structures may limit the application of this technique. To solve this problem, we use the natural frequencies of closed-loop systems with virtual passive controllers [8], which resemble mass–spring dashpots. When closed-loop systems are used for damage detection, one important issue is how to design controllers that can enhance the sensitivity of the identified natural frequencies to structural damage. In this paper, we plan to apply not only passive controllers but also low-order and fixed-structure controllers to structural damage detection. Passive controllers almost always augment the damping of the system and maintain the stability of the system. Low-order and fixed-structure controllers can be designed to incorporate modern ideas of robustness and optimality.

The correlation approach has been used to study the characteristics of the transfer function parameter change due to structural damage [5]. This study shows that the correlation method can generate effective neural network (NN) patterns and significantly simplify the NN structural damage detection procedures. This paper presents a novel study of an optimal controller design for structural damage detection. The proposed optimal control technique is based on a neural network approach [5], which uses the pattern of identified natural frequency correlation to distinguish the damaged element from the undamaged element. The locations of sensors and actuators are important for the performance of vibration suppression as well as the performance of structural damage detection. The associated issue of sensor placement will also be discussed.

2. Feedback controller

The study is based on closed-loop systems with different types of feedback controllers, passive controllers and low-order controllers.

2.1. Passive controllers

The second-order dynamic equation of structural vibration is used,

$$M\ddot{x} + D\dot{x} + Kx = Bu, \tag{1}$$

$$y = C_a\ddot{x} + C_v\dot{x} + C_d x. \tag{2}$$

Here x is a $p \times 1$ displacement vector, and M, D, and K are mass, damping, and stiffness matrices, respectively. In the measurement equation, y is the $q \times 1$ measurement vector, and C_a, C_v and C_d are acceleration, velocity, and displacement influence matrices. The natural frequency vector of the open-loop system is defined as

$$\omega_0 = [\omega_1 \omega_2 \ldots \omega_p]^{\mathrm{T}}. \tag{3}$$

The measurement equation may be used either directly or indirectly for a feedback controller design. First we use direct output feedback, where the input vector u is

$$u = -Fy = -FC_a\ddot{x} - FC_v\dot{x} - FC_d x, \tag{4}$$

where F is a constant gain matrix. Substituting Eq. (4) into Eq. (1) yields

$$(M + BFC_a)\ddot{x} + (D + BFC_v)\dot{x} + (K + BFC_d)x = 0. \tag{5}$$

The natural frequency vectors of the m closed-loop systems with different designed gain matrices are used for damage detection. These vectors are computed as

$$\omega_j = [\omega^j_{c1}\omega^j_{c2} \ldots \omega^j_{cp}]^{\mathrm{T}}, \quad j = 1, \ldots, m,$$

where ω^j_{ci} is the ith natural frequency of the jth closed-loop system.

Second, the feedback controller is described as a set of second-order dynamic equations,

$$M_c\ddot{x}_c + D_c\dot{x}_c + K_c x_c = B_c u_c, \tag{6}$$

$$y_c = C_{ac}\ddot{x}_c + C_{vc}\dot{x}_c + C_{dc}x_c. \tag{7}$$

Here x_c is the controller state vector, and M_c, D_c, and K_c are the controller mass, damping and stiffness matrices, respectively. The quantities $M_c, D_c, K_c, C_{ac}, C_{vc}$, and C_{dc} are the design parameters for the controller. Let the input vectors u and u_c [9] be

$$u = y_c = C_{ac}\ddot{x}_c + C_{vc}\dot{x}_c + C_{dc}x_c, \tag{8}$$

$$u_c = y = C_a\ddot{x} + C_v\dot{x} + C_d x. \tag{9}$$

Substituting Eq. (8) into Eq. (1) and Eq. (9) into Eq. (6) yields

$$M_t\ddot{x}_t + D_t\dot{x}_t + K_t x_t = 0, \tag{10}$$

where

$$M_t = \begin{bmatrix} M & -BC_{ac} \\ -B_cC_a & M_c \end{bmatrix}, \quad D_t = \begin{bmatrix} D & -BC_{vc} \\ -B_cC_v & D_c \end{bmatrix}, \tag{11}$$

$$K_t = \begin{bmatrix} K & -BC_{dc} \\ -B_cC_d & K_c \end{bmatrix}, \quad x_t = \begin{bmatrix} x \\ x_c \end{bmatrix}. \tag{12}$$

In the controller design, $M_c, D_c, K_c, C_{ac}, C_{dc}$, and C_{vc} are chosen such that the closed-loop system is stable [9,10].

2.2. Low-order controllers

Low-order controllers are described as the form of a transfer function. The closed-loop transfer function of a open-loop system $G(s)$ with a negative feedback controller $K(s)$ is

$$P(s) = \frac{G(s)}{I + G(s)K(s)}. \tag{13}$$

For damage detection, we use the identified natural frequencies of m closed-loop systems with different designed controllers.

3. Correlation approach

A brief introduction to the correlation approach is given in this section. The changes of the parameter vectors of the ith referred damage case [5], such as the stiffiness loss of the ith element, are defined as

$$\Delta\omega_{ij} = \omega_{ij} - \omega_j, \quad j = 0, 1, \ldots, m, \tag{14}$$

where ω_{ij} are the natural frequency vectors of the ith damage case. The change vectors $\Delta\omega_{ij}$ are used as references and these vectors are obtained from analytical models such as finite element models. Because of the perturbation of each parameter, we define the weighted change vectors [3] as

$$\Delta\omega_{ij}^W = [\Delta\omega_{ij}(1)/W_{j1} \ldots \Delta\omega_{ij}(k)/W_{jk}]^{\mathrm{T}}, \tag{15}$$

where $\Delta\omega_{ij}(l)$ is the lth element of $\Delta\omega_{ij}$, and W_{jl} is the standard deviation of $\{\Delta\omega_{1j}(l), \Delta\omega_{2j}(l), \ldots, \Delta\omega_{nj}(l)\}$ for the considered n damage cases. The correlations between the tested system with the weighted change vectors $\Delta\omega_j^W$, which represent the difference of the identified parameters between the tested system and the healthy system, and the ith damage case are defined as

$$C_{ij} = \frac{(\Delta\omega_j^W)^{\mathrm{T}}\Delta\omega_{ij}^W}{|\Delta\omega_j^W||\Delta\omega_{ij}^W|}, \quad j = 0, 1, \ldots, m. \tag{16}$$

The correlation C_{ij} represents the cosine between two vectors. The value of the correlation C_{ij} is between -1 and 1. When C_{ij} is less than 0, the change vector $\Delta\omega_j^W$ of the tested system is in a different direction ($>90^\circ$) from the change vector $\Delta\omega_{ij}^W$ due to the ith element damage. It strongly implies that the ith element is not damaged [3]. The minimum correlation of the tested system corresponding to the ith damage case is defined as

$$C_i = \min\{C_{i0}, C_{i1}, \ldots, C_{im}\}. \tag{17}$$

The magnitude ratios between the tested system and the ith damage case are defined as

$$R_{ij} = \frac{|\Delta\omega_j^W|}{|\Delta\omega_{ij}^W|}, \quad j = 0, 1, \ldots, m. \tag{18}$$

The ratio between the maximum magnitude ratio and minimum ratio will be used as an index for damage detection, and it is defined as

$$RA_i = \frac{RX_i}{RN_i}, \tag{19}$$

where

$$RX_i = \max\{R_{i0}, \ldots, R_{im}\}, \quad RN_i = \min\{R_{i0}, \ldots, R_{im}\}. \tag{20}$$

The magnitude ratios can be used to identify the intensity and location of damage [6].

4. Neural network approach

In this section, a neural network approach based on the correlation approach is presented. Fig. 1 shows the diagram of this NN method with Perceptron architecture [11]. The inputs of the network are the identified change vectors $\Delta\omega_j^W$, $j = 0, 1, \ldots, m$ and the identified magnitude ratios RA_i, $i = 1, \ldots, n$. The input vector $\mathbf{p}$ of the networks is defined as

$$\mathbf{p} = \left[\frac{(\Delta\omega_0^W)^\mathrm{T}}{|\Delta\omega_0^W|} \cdots \frac{(\Delta\omega_m^W)^\mathrm{T}}{|\Delta\omega_m^W|} - RA_1 \ldots - RA_n\right]^\mathrm{T}. \tag{21}$$

For damage detection, the outputs of the first layer are used to indicate the damage status determined by each of the variables C_{ij} and RA_i. The elements of the output vector $\mathbf{a}^1$ are

$$\begin{aligned}
a_1^1 &= h(C_{10} + b_2^1)\\
&\vdots\\
a_{m+1}^1 &= h(C_{1m} + b_{m+1}^1)\\
a_{m+2}^1 &= h(-RA_1 + b_{m+2}^1)\\
&\vdots\\
a_{n(m+2)-1}^1 &= h(C_{nm} + b_{n(m+2)-1}^1)\\
a_{n(m+2)}^1 &= h(-RA_n + b_{n(m+2)}^1),
\end{aligned} \tag{22}$$

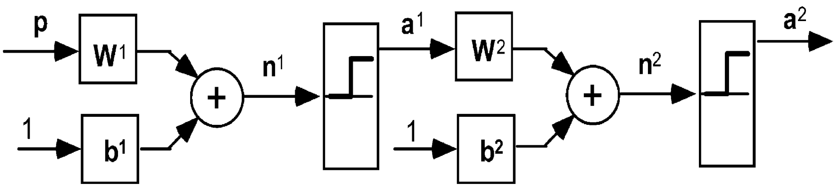

Fig. 1. Two-layer Perceptron network.

where h is the *hard limit function* [11] and it is defined as

$$h(x) = \begin{cases} 0, & x<0, \\ 1, & x \geqslant 0. \end{cases} \tag{23}$$

The weight matrix of the first layer is computed as

$$W^1 = \begin{bmatrix} \frac{(\Delta\omega_{10}^W)^{\mathrm{T}}}{|\Delta\omega_{10}^W|} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \frac{(\Delta\omega_{11}^W)^{\mathrm{T}}}{|\Delta\omega_{11}^W|} & \cdots & 0 & 0 & \cdots & \vdots \\ \vdots & \vdots & & & & & \vdots \\ 0 & 0 & \cdots & \frac{(\Delta\omega_{1m}^W)^{\mathrm{T}}}{|\Delta\omega_{1m}^W|} & 0 & \cdots & \\ 0 & 0 & \cdots & 0 & 1 & \cdots & 0 \\ \frac{(\Delta\omega_{20}^W)^{\mathrm{T}}}{|\Delta\omega_{20}^W|} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & & & & \vdots \\ \frac{(\Delta\omega_{n0}^W)^{\mathrm{T}}}{|\Delta\omega_{n0}^W|} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & & & & \vdots \\ 0 & 0 & \cdots & \frac{(\Delta\omega_{nm}^W)^{\mathrm{T}}}{|\Delta\omega_{nm}^W|} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 1 \end{bmatrix}. \tag{24}$$

The components of this weight matrix can be trained with the use of the updated model and the identified transfer function parameters. The elements of the bias vector b^1 are chosen to distinguish damage status. For example, if b_1^1 is chosen as -0.99, the "yes" damage status of the first element from the correlation C_{10} requires that C_{10} be larger than 0.99 ($a_1^1 = 1$). The damage status of element 1 is determined by correlations $C_{1j}, j = 0, 1, \ldots, m$ and the ratio RA_1. When all the outputs $a_i^1,\ i = 1, \ldots, m+2$ are 1, then a_1^2 is 1 and element 1 is a possible damaged element. The ith element of the output vector $\mathbf{a}^2$ is used to indicate the damage status of the ith element. The elements a_i^2 are computed as

$$\begin{aligned} a_1^2 &= h(a_1^1 + a_2^1 + \cdots + a_{m+2}^1 + b_1^2) \\ a_2^2 &= h(a_{m+3}^1 + a_{m+4}^1 + \cdots + a_{2(m+2)}^1 + b_2^2) \\ &\vdots \\ a_n^2 &= h(a_{(n-1)(m+2)+1}^1 + \cdots + a_{n(m+2)}^1 + b_n^2) \end{aligned} \tag{25}$$

with

$$b_i^2 = -m - 2, \quad i = 1, \ldots, n. \tag{26}$$

The element a_i^2 is 1, which indicates that the ith element is the possible damaged element, if all $a^1_{(i-1)(m+2)+j}$, $j = 1, \ldots, m+2$, which correspond to the ith damage case, are equal to 1. To demonstrate the results of damage detection, the following term is defined for discussion.

Identifiable damage: When the ith element is damaged, this damage case is identifiable if $a_i^2 = 1$ and $a_j^2 = 0$, $j = 1, \ldots, i-1, i+1, \ldots, n$.

If the damage of the ith element is identifiable, then the damage of the ith element can be uniquely identified. The damage of element i is identifiable if and only if

$$a_i^2 = 1, \tag{27}$$

and

$$a_d = \sum_{i=1}^{n} a_i^2 = 1. \tag{28}$$

The variable a_d is the number of possible damage candidates.

5. Optimal controller design

The problem is to find an "optimal" controller that enhances the difference of correlations and magnitude ratios between the damaged element and the undamaged element, so the damaged element can be distinguished. In this paper, the objective function is based on the minimum correlations C_i and ratios RA_i for all possible damage cases. For passive controller design, the function related to the ith element damage case can be defined as

$$\begin{aligned} f_i = e_1 \sum_{j=1, j\neq i}^{n} a_j^2 + e_2 \sum_{j=1, j\neq i}^{n} (C_j - b_1) h(C_j - b_1) \\ + e_3 \sum_{j=1, j\neq i}^{n} (b_2 - RA_j) h(b_2 - RA_j), \end{aligned} \tag{29}$$

where e_j are given constants, b_1 is chosen to distinguish damage based on correlation, b_2 is chosen to distinguish damage based on magnitude ratio, and h is the *hard limit function*. For the function f_i, the first term results from the number of damage candidates excluding the ith element, the second term is from the correlation of the element with minimum correlation higher than the specified value b_1, and the third term is from ratios lower than the specified value b_2. The objective function for optimal controller is defined as

$$f = \sum_{i=1}^{n} f_i. \tag{30}$$

6. Results and discussion

The finite element model of a cantilevered aluminum Euler's beam, as shown in Fig. 2, is used for study. The length, width, and thickness of this beam are 1, 0.0254, and 0.000635 meters, respectively. The study is based on the analysis of the finite element model of this beam structure [12]. For structural damage, we consider the stiffness loss of 15 elements with equal lengths from the fixed end to the free end. Next, we examine the results for each element with 20% stiffness loss. The referred vectors $\Delta\omega_{ij}^{W}$, which are used for computing correlations and magnitude ratios, are the weighted parameter changes due to 0.1% stiffness loss of the ith element. The previous works show that correlations and magnitude ratios have negligible changes with different levels of damage, such as stiffness loss.

6.1. Direct output feedback

In the direct output feedback example, we use two displacement measurements located at positions 3 and 15, respectively. The first closed-loop system has the collocated output feedback controller at position 3, where the controller gain is g_1. The second closed-loop system has the collocated output feedback controller at position 15, where the controller gain is g_2. These collocated output feedback controllers function like stiffness added at positions 3 and 15:

$$\begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \\ 0 & \cdots & k_3 + g_1 & \cdots & \vdots \\ \vdots & & \vdots & \ddots & \\ 0 & & \cdots & & k_{15} + g_2 \end{bmatrix}. \tag{31}$$

The natural frequencies of the first 3 modes of the open-loop system and the two closed-loop systems are used for damage detection. Table 1 lists the natural frequencies of the first 3 modes of the open-loop system for three damage cases (20% stiffness loss of element 1, 3, or 5). Fig. 3 shows the correlation for the element 3 damage case when various controller gains are applied to the system, where g_1 and g_2 are gains of output feedback at positions 3 and 15, respectively. Fig. 3(a) shows the results related to the first closed-loop system, the increase of the gain at position 3 can dramatically enhance the correlation difference between the first 2 elements and element 3,

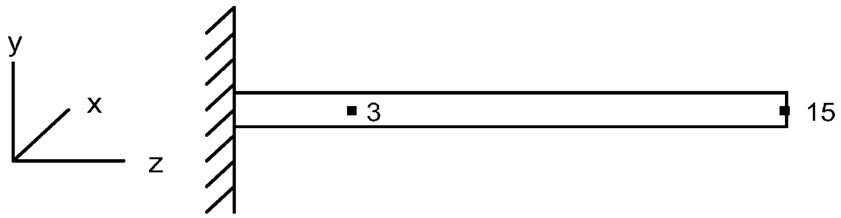

Fig. 2. Cantilevered Euler's beam.

Table 1
Natural frequencies (Hz) of various damage cases

Mode	No damage	Element 1	Element 3	Element 5
1	0.2314	0.2246	0.2269	0.2287
2	1.4571	1.4252	1.4546	1.4523
3	4.1401	4.0700	4.1322	4.0853

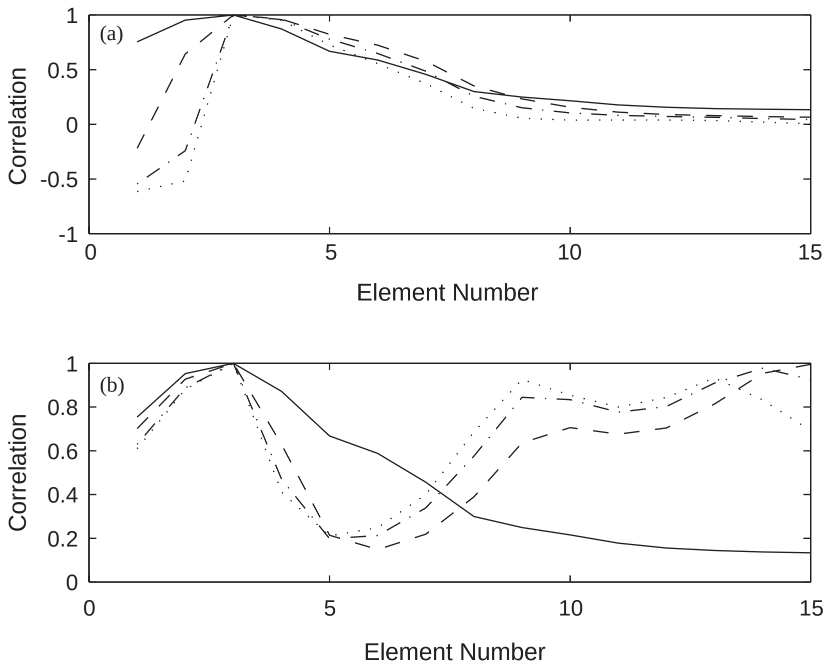

Fig. 3. Correlation for element 3 damage case: (a) —, $g_1 = 0$; - -, $g_1 = 0.5$; - · -, $g_1 = 1$; · · ·, $g_1 = 1.5$. (b) —, $g_2 = 0$; - -, $g_2 = 0.5$; -·-, $g_2 = 1$; · · ·, $g_2 = 1.5$.

where the correlation is 1 for element 3. The increase of the gain related to the displacement output at position 15 (as shown in Fig. 3(b)) can dramatically enhance the correlation difference between the first 5 (exclude element 3) elements and element 3. But the increase of gain does not always enhance the correlation difference between the damaged element and the undamaged element. Fig. 3 shows that all the correlations C_{3j}, corresponding to the element 3 damage, are 1. Also the ratio RA_3 is 1 for the open-loop and the closed-loop systems with different controllers. In this example, the variables b_i^1 corresponding to correlation are chosen as -0.97, and the variables b_i^1 corresponding to magnitude ratio are chosen as 1.2. This implies that if and only if $C_{ij} \geqslant 0.97$, $j = 0, 1, \ldots, m$, and $RA_i \leqslant 1.2$, the output a_i^2 is 1. If the output a_i^2 is 1, the ith element is a damage candidate.

To obtain optimal controllers, we use function *f min* in *Matlab* to find the minimum of the defined objective function in Eq. (30). The constant variables in Eq. (29) are chosen as $e_1 = 0.03$,

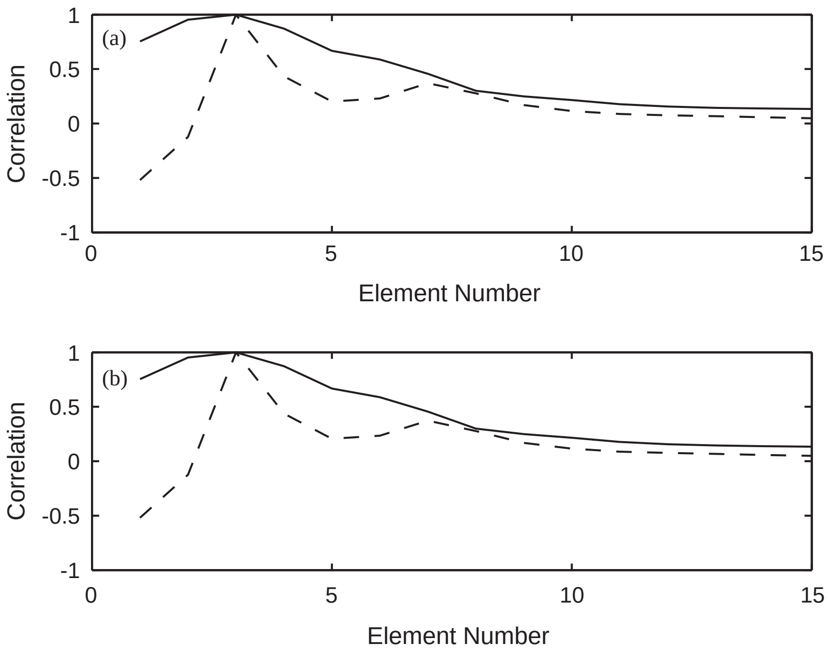

Fig. 4. Correlation for element 3 damage case with optimal controller: —, correlation of open-loop system; - -, minimum correlation. (a) Tested system with 0.1% stiffness loss, (b) tested system with 20% stiffness loss.

Table 2
Number of possible damage candidates a_d

Element	Open	$g_1 = 1$ $g_2 = 1$	$g_1 = 0.900$ $g_2 = 1.250$
1	1	1	1
2	1	1	1
3	1	1	1
4	1	1	1
5	1	1	1
6	1	1	1
7	2	1	1
8	1	1	1
9	2	1	1
10	1	1	1
11	2	2	2
12	5	3	2
13	4	2	1
14	4	1	1
15	4	1	1

$e_2 = 1$, $e_3 = 0$, $b_1 = 0.97$, $b_2 = 1.2$. This is a nonlinear optimization problem, the initial gains are chosen as $g_1 = 1$, $g_2 = 1$. The optimal gains are $g_1 = 0.900$, $g_2 = 1.250$, respectively. Fig. 4 shows the correlation for element 3 with stiffness loss of 0.1%, and 20% when the natural frequencies of the open-loop system and the two closed-loop systems with optimal controllers are used. The use of the closed-loop systems with optimal controllers can dramatically enhance the correlation difference between element 3 (damaged element) and other elements with correlation (open-loop system) close to 1. The correlation of the open-loop system and minimum correlation have negligible changes when stiffness loss varies from 0.1% to 20%. Table 2 shows the results when the natural frequencies of the open-loop system and the two closed-loop systems with various gains are used. For the open-loop system, 7 damaged cases are not identifiable, where a_d is larger than 1. For the element 12 damage case, there are 5 damage candidates ($a_d = 5$). The number of non-identifiable damage cases reduces to 2 when the correlations of open-loop system and two closed-loop systems with optimal controllers are used. For the systems with optimal controllers, only two damage cases (elements 12 and 13) cannot be distinguished from each other.

6.2. Controller with second-order dynamics

Two passive controllers, which are spring–mass systems with two-degrees-of-freedom (Fig. 5), are attached at positions 3 and 15 of the cantilevered Euler's beam. The dynamic equations of these two passive controllers are

$$\begin{bmatrix} m_{c1} & 0 \\ 0 & m_{c2} \end{bmatrix}\begin{bmatrix} \ddot{x}_{c1} \\ \ddot{x}_{c2} \end{bmatrix} + \begin{bmatrix} k_{c1}+k_{c2} & -k_{c2} \\ -k_{c2} & k_{c2} \end{bmatrix}\begin{bmatrix} x_{c1} \\ x_{c2} \end{bmatrix} = \begin{bmatrix} k_{c1} \\ 0 \end{bmatrix} x_3, \tag{32}$$

$$\begin{bmatrix} m_{c3} & 0 \\ 0 & m_{c4} \end{bmatrix}\begin{bmatrix} \ddot{x}_{c3} \\ \ddot{x}_{c4} \end{bmatrix} + \begin{bmatrix} k_{c3}+k_{c4} & -k_{c4} \\ -k_{c4} & k_{c4} \end{bmatrix}\begin{bmatrix} x_{c3} \\ x_{c4} \end{bmatrix} = \begin{bmatrix} k_{c3} \\ 0 \end{bmatrix} x_{15}, \tag{33}$$

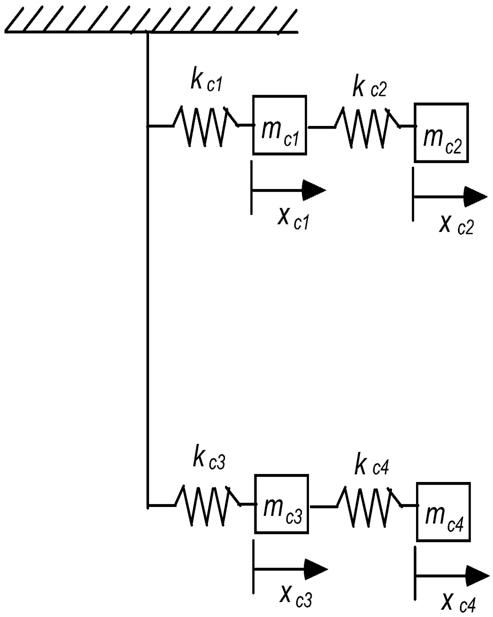

Fig. 5. Cantilevered Euler's beam with passive dynamic controllers.

Table 3
Design variables of controllers

	Controller 1	Controller 2
m_{c1}	0.002	0.008
m_{c2}	0.004	0.006
m_{c3}	0.006	0.004
m_{c4}	0.008	0.002
k_{c1}	g_1	$2g_2$
k_{c2}	$2g_1$	g_2
k_{c3}	$2g_1$	g_2
k_{c4}	g_1	$2g_2$

Table 4
Number of possible damage candidates a_d

Element	$g_1 = 0.2$ $g_2 = 0.2$	$g_1 = 1$ $g_2 = 1$	$g_1 = 1.239$ $g_2 = 2.222$
1	1	2	1
2	1	1	1
3	1	1	1
4	1	1	1
5	1	1	1
6	1	2	1
7	1	1	1
8	1	2	1
9	2	3	1
10	3	3	1
11	3	2	1
12	3	2	1
13	3	1	1
14	3	1	1
15	2	1	1

where x_3 and x_{15} are displacements at positions 3 and 15, respectively. The results of damage detection are based on the analysis of the first 3 modes of two closed-loop systems with controllers that have design variables as listed in Table 3. The mass variables of both controllers are chosen as constant. The stiffness variables of both controllers are

$$[k_{c1}\, k_{c2}\, k_{c3}\, k_{c4}] = [1\,2\,2\,1]g_1, \quad [k_{c1}\, k_{c2}\, k_{c3}\, k_{c4}] = [2\,1\,1\,2]g_2.$$

Each controller has one varied gain g_i. The initial gains for optimization process are $g_1 = 1$, $g_2 = 1$. The optimal gains are $g_1 = 1.239$, $g_2 = 2.222$, respectively.

Table 4 shows the results when the natural frequencies of the open-loop system and the two closed-loop systems with various gains are used. For the systems with $g_1 = 0.2$, $g_2 = 0.2$, seven damaged cases are not identifiable. For the systems with optimal controllers, all the damage cases are identifiable.

6.3. Low-order controllers

For the low-order controllers, the collocated output feedback controller with displacement measurement at each node is used. The transfer function of the designed controller is

$$\frac{k_2 s + k_3}{s + k_1}.$$

There are 3 variables for this controller. In this optimal control design, only natural frequency correlations excluding magnitude ratios are used. The objective function related to the ith damage case is defined as

$$f_i = e_1 \sum_{j=1, j \neq i}^{n} a_j^2 + e_2 \sum_{j=1, j \neq i}^{n} (C_j - b_1) h(C_j - b_1) + f_i^c.$$

For function f_i, the first term is from the number of damage candidates excluding the ith element, the second term is from the correlation of the element with correlation higher than the specified value b_1, and f_i^c is related to the specified controller performances, such as damping and controller force. The optimal controller is designed to: (1) enhance the correlation difference between the damaged element and the undamaged element for all the damage cases; (2) maintain the stability of the controller ($k_1 > 0$); (3) satisfy the specified damping performance; (4) restrict the control force.

The natural frequencies of the first 3 modes of the open-loop system and one closed-loop system with a collocated feedback controller at each position are used for damage detection. Table 5 shows the results when the natural frequencies of the open-loop and the closed-loop system with optimal controller at various positions n_s (node number in Fig. 2) are used.

Table 5
Number of possible damage candidates a_d

Element	$n_s = 1$	$n_s = 8$	$n_s = 11$	$n_s = 15$
1	1	1	1	1
2	1	1	1	1
3	1	1	1	1
4	1	1	1	1
5	1	1	1	1
6	1	1	1	1
7	2	1	1	1
8	1	1	1	1
9	2	1	2	1
10	1	1	1	1
11	2	2	2	1
12	5	5	3	1
13	4	4	3	1
14	4	4	4	1
15	4	4	2	1

From Table 5, when the natural frequencies of the open-loop and the closed-loop system with optimal controller at position 1 are used, 7 damaged cases are not identifiable, where a_d is larger than 1. For the element 12 damage case, there are 5 damage candidates ($a_d = 5$). All the damage cases are identifiable when the natural frequencies of the open-loop and the closed-loop system with optimal controller at position 15 are used. The performance of damage detection is very sensitive to sensor/actuator location. For this example, the optimal sensor/actuator position is at position 15. In this paper, the natural frequencies of the first three modes are used for damage detection. In general, the low-frequency modes dominate structural vibration, and they are less sensitive to noise. The effect of number of modes on damage detection can be found in Ref. [5].

From the results with different types of optimal controllers, the design based on the low-order controller provides the best performance for damage detection. This performance cannot be achieved with the designed optimal passive controllers. In this paper, the finite element model of a flexible beam with 20% stiffness loss of each element is used as the tested system. In the real application, the identified transfer functions of the tested system are used for damage detection [6]. The effect of the identified parameter uncertainty on structural damage detection can be found in Refs. [3,5]. The correlation approach has been applied to a truss structure [3]. A more efficient correlation approach based on the NN technology with the trained patterns can be applied to a large more complex structure.

7. Conclusions

This paper presents a novel study of optimal controller design for structural damage detection. This study is based on a neural network approach that uses the correlation of the identified natural frequency change of open-loop and closed-loop systems. In the optimal control designs, passive controllers and low-order controllers are used. The results show that the use of optimal controllers can significantly enhance the correlation difference between the damaged element and the undamaged element. This can dramatically improve the performance of damage detection. The example of low-order controllers demonstrates that the controller can be designed for both the performance of structural damage detection and also the specified damping performance. The performance of damage detection is very sensitive to sensor/actuator location.

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References

[1] S.W. Doebling, C.R. Farrar, M.B. Prime, A summary review of vibration-based identification methods, *Shock and Vibration Digest* 205 (1998) 631–645.

[2] A.K. Pandey, M. Biswas, M. Samman, Damage detection from changes in curvature mode shapes, *Journal of Sound and Vibration* 145 (1991) 321–332.
[3] J.-S. Lew, Using transfer function parameter changes for damage detection of structures, *AIAA Journal* 33 (1995) 2189–2193.
[4] D.C. Zimmerman, M. Kaouk, Structural damage detection using a subspace rotation algorithm, *Proceedings of the 33th AIAA/ASME/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, AIAA, Washington, DC, 1992, pp. 2341–2350.
[5] J.-S. Lew, Damage detection using neural networks and transfer function correlation, *Proceedings of the 18th International Modal Analysis Conference*, San Antonio, TX, 2000, pp. 1783–1789.
[6] J.-S. Lew, C. Hyde, M. Wade, Damage detection of flexible beams using transfer function correlation, *Proceedings of the ASME 17th Biennial Conference on Mechanical Vibration and Noise*, Las Vegas, September 12–15, 1999.
[7] P. Trivailo, L.A. Plotnikova, L.A. Wood, Enhanced parameter identification for damage detection and structural integrity assessment using "Twin" structures, *Fifth International Congress on Sound and Vibration*, University of Adelaide, SA, 1997, pp. 1733–1741.
[8] J.-S. Lew, J.-N. Juang, Structural damage detection using virtual passive controllers, *Journal of Guidance, Control, and Dynamics* 25 (2002) 419–424.
[9] J.N. Juang, M. Phan, Robust controller designs for second-order dynamic systems: a virtual passive approach, *Journal of Guidance, Control and Dynamics* 15 (1992) 1192–1198.
[10] A.M. Bruner, W.K. Belvin, L.G. Horta, J.N. Juang, Active vibration absorber for the CSI evolutionary model design and experimental results, *Journal of Guidance, Control and Dynamics* 15 (1992) 1253–1257.
[11] M.T. Hagan, H.B. Demuth, M. Beale, *Neural Network Design*, PWS Publishing Company, Boston, 1996.
[12] R.R. Craig, *Structural Dynamics: An Introduction to Computer Methods*, Wiley, New York, 1981.